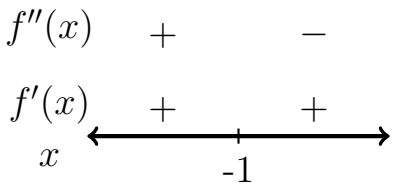
Quiz Solution

March 30, 2018

1. (2 points) Sketch a single graph satisfying all of the following:

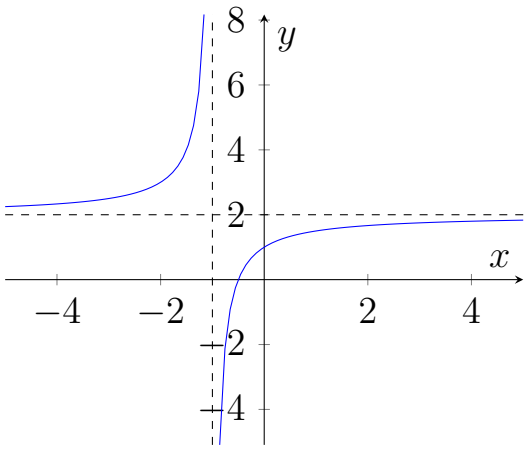
   - $f(x)$ has asymptotes $x = -1$ and $y = 2$
   - $f'(x) > 0$ on the intervals $(-\infty, -1) \cup (-1, \infty)$
   - $f''(x) > 0$ on the intervals $(-\infty, -1)$
   - $f''(x) < 0$ on the interval $(-1, \infty)$

   Solution: We have the following sign charts:

   So $f(x)$ is increasing and concave up on $(-\infty, -1)$ and increasing and concave down on $(-1, \infty)$.

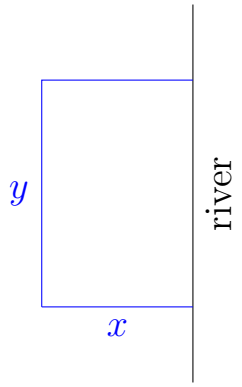
   First graph the asymptotes, then sketch the graph. Here is one possible graph of $f(x)$:

   Answer: See above.

2. (2 points) You are building a rectangular pasture along a straight river. Three sides of fencing are needed, and the fourth side will be bounded by the river. If you have 1,000 feet of fencing, what is the largest possible area of the pasture?

   Solution: We have the following picture:



We are trying to maximize $A = xy$ subject to the constraint $2x + y = 1000$.

Solving for y, we get $y = 1000 - 2x$.

Substituting into A, we get $A(x) = x(1000 - 2x) = 1000x - 2x^2$.

Taking the derivative, we want to solve $A' = 1000 - 4x \stackrel{\text{set}}{=} 0$. Solving, $x = 250$.

We can check that this value of x maximizes the area using the second derivative test: $A''(x) = -4 < 0$ for all x, so we have a relative maximum at $x = 250$.

Finally, the maximum area is $A(250) = 250(1000 - 2(250)) = 125000$

Answer: $125,000$ square feet

3. (1 point) What is your favorite season?

   Answer: Answers will vary.